

## STRESSES IN A LAYER OF A HIGH-CONCENTRATION DISPERSE SYSTEM UNDER DRYING CONDITIONS

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*A mathematical model of the stress-strain state of a high-concentration disperse system in drying is proposed. The rheological behavior of the system in the saturation region is evaluated in terms of the micromechanics of approach of particles with account for viscous fluid flow in interparticle gaps and surface forces.*

Various physicochemical processes (temperature and moisture changes, chemical reactions, phase transitions) may cause bulk deformations of particles in a medium in the absence of mechanical forces. Nonuniform distribution of such natural deformations, with the continuity of the material being maintained, results in natural stresses [1]. The strains and stresses exert an inverse influence on the physicochemical processes, thus leading to the problem on interrelated fields [2].

In drying, free shrinkage of the material is responsible for natural stresses. In [3], based on the thermodynamics of irreversible processes equations of heat and mass transfer of nonlinearly elastic and elastoplastic solids are proposed. In [4], the problem of natural stresses and strains in the drying of a thin plate is analyzed. The approach adopted in [3, 4] uses the theory of small elastoplastic strains [5]. Here, the shear modulus, which depends on the strain intensity, is brought out from under the differential sign with respect to the space coordinates and thus becomes, in actuality, a constant quantity. Therefore the results of [3, 4] are valid only for a linearly elastic body. The problem of plate drying [4] is solved for given "free moisture-heat strains," which is mathematically equivalent to the well-known solution of the thermoelasticity problem for a given temperature distribution [6]. The development of the stress-strain state in intense drying of a plate made of an ideally elastoplastic material is considered in [7] where the distribution of residual stresses over the thickness of the plate after its complete drying is determined. The rheological behavior of a material in moist and dry zones is, generally speaking, different. This is taken into account in [8], where the region of complete saturation is described by the Kelvin–Voigt viscoelastic model, and the dry zone by Hooke's law.

**1. Mathematical Model.** We shall consider drying of a layer of a high-concentration disperse system of thickness  $2h$ . In drying, the behavior of disperse materials has distinctive features due to the presence of various contacts between particles of the solid phase. In a moist zone, the particles interact via a liquid interlayer, thus imparting viscous properties to the system. In the course of drying such a free disperse system (solid phase-liquid) turns into a bound disperse system with direct (through condensation) contacts between particles, and therefore the rheological behavior of the medium in moist and dry zones is different.

We assume the drying regime to be so intense that a narrow evaporation zone develops and separates the layer into a complete saturation region and a dry region. Henceforth the dimensions of the evaporation zone are neglected and it is modeled by an evaporation front. Such a scheme makes it possible to trace comparatively simply the development of natural stresses in the material and to evaluate them.

A material undergoes shrinkage in both the saturation and evaporation zones. In the former, particles preliminarily approach each other under the action of contracting forces, thus changing the structure of the system, and, as a consequence, the medium is subjected to the action of capillary forces in the evaporation zone. The corresponding shrinkage is determined by the distribution and duration of these forces and, ultimately, by a velocity

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of the evaporation front. For rapid drying the vapor flow  $j$  is inversely proportional to the width of the dry zone [9] and is approximately equal to:

$$j = \frac{[D]MP_s}{RT(h - z^0)},$$

where  $z^0$  is the coordinate of the evaporation front reckoned from the middle plane of the layer. Then the velocity of the evaporation front is

$$V = -\frac{dz^0}{dt} = \frac{j}{\rho} = \frac{A}{(1 - z^0/h)}, \quad (1)$$

where  $A = [D]MP_s/(RT) = h/(t_1)$  is a constant. Hence  $\zeta^0 = 1 - z^0/h = (2At/h)^{1/2}$ .

The rheological behavior of a dry material may be described by an elasticity law with account for shrinkage [1]:

$$\sigma_{ij} = \frac{E}{1 + \nu} [(\varepsilon_{ij} - \varepsilon_{ij}^0) + \frac{\nu}{1 - 2\nu} \delta_{ij} (\varepsilon_{kk} - \varepsilon_{kk}^0)]. \quad (2)$$

We now superpose the plane  $(x, y)$  of a Cartesian coordinate system and the middle plane of a layer of a drying medium. For a thin layer, with variation in the moisture content only over the thickness, we may assume, by analogy with the thermoelastic case [6], that

$$\sigma_{xy} = \sigma_{zz} = \sigma_{xz} = \sigma_{yz} = 0, \quad \sigma_{xx} = \sigma_{yy} = \sigma(z), \quad (3)$$

$$\varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0, \quad \varepsilon_{zz} \neq 0, \quad \varepsilon_{xx} = \varepsilon_{yy} = \varepsilon(z).$$

Therefore relations (2) with account for (3) may be represented in the form

$$\begin{aligned} \sigma(z) &= \frac{E}{1 + \nu} \left[ (\varepsilon - \varepsilon^0) + \frac{\nu}{1 - 2\nu} (\varepsilon_{kk} - \varepsilon_{kk}^0) \right], \\ \sigma_{zz} &= \frac{E}{1 + \nu} \left[ (\varepsilon_{zz} - \varepsilon_{zz}^0) + \frac{\nu}{1 - 2\nu} (\varepsilon_{kk} - \varepsilon_{kk}^0) \right] = 0. \end{aligned} \quad (4)$$

Eliminating  $\varepsilon_z$  from (4) and using the last relation, we arrive at

$$\sigma(z, t) = \frac{E}{1 - \nu} [\varepsilon(t) - \varepsilon^0], \quad (5)$$

where  $\varepsilon^0 - \varepsilon_{xx}^0 = \varepsilon_{yy}^0$  is the shrinkage.

From the conditions of simultaneous strains with account for (3) the single equation  $\partial^2 \varepsilon / \partial z^2 = 0$  remains, whose solution is

$$\varepsilon(z, t) = a(t)z + b(t),$$

and for the symmetric problem, using the condition  $(\partial \varepsilon / \partial z)_{z=0} = 0$  at the layer's center we obtain that the strain is constant over the layer thickness and is a function of time or the position of the evaporation front:

$$\varepsilon = \varepsilon(t), \quad \varepsilon = \varepsilon(\zeta^0). \quad (6)$$

Shrinkage  $\varepsilon^0$  of a material particle with coordinate  $\zeta$  develops at the moment when the evaporation front passes through it. A portion of the shrinkage is equal to the strain at this moment:  $\varepsilon(\zeta) = \varepsilon(\zeta^0 = \zeta)$ . The value of  $\varepsilon(\zeta)$  determines the structure of the system when the latter enters the evaporation zone. The action of capillary

forces in this zone determines the remaining shrinkage  $\varepsilon^e$ . It depends on both the initial structure ( $\varepsilon(\zeta)$ ) and the drying regime (the velocity of the evaporation front):  $\varepsilon^e(\varepsilon(\zeta), V)$ . Thus, the shrinkage may be represented as the sum

$$\varepsilon^0(\zeta) = \varepsilon(\zeta) + \varepsilon^e. \quad (7)$$

The rheological behavior of a disperse system in the saturated region may be described in terms of the micromechanics of approach of particles. It is determined by two main factors, namely, viscous flow of the liquid in an interparticle gap and surface forces. These forces prevent particles from approaching each other and depend on an interlayer thickness. We will model a macropoint of the medium by a chain (ensemble) of spherical particles. We assume that the particle sizes at the macropoint are distributed in accordance with a Gaussian law with the density

$$\varphi(R) = \frac{1}{(2\pi\langle\sigma\rangle^2)^{1/2}} \exp\left[-\frac{(R - \langle R \rangle)^2}{2\langle\sigma\rangle^2}\right],$$

where  $\langle\sigma\rangle^2$  is the dispersion;  $\langle R \rangle$  is the mean particle radius.

We now single out one of the links of the chain, i.e., two particles of radius  $R$ , and write the equation of their approach with neglect of inertial forces:

$$F_e(t) + F(H) + F_f(\dot{H}, H) = 0. \quad (8)$$

Here  $H$  is the shortest distance between the particles;  $\dot{H} \equiv \partial H / \partial t$ ;  $F_e(t) = \langle S \rangle \sigma^y(t)$  is the force determined by the macrostress  $\sigma^y$  in the saturated region;  $\langle S \rangle = \pi \langle R \rangle^2$ . The force is the same for pairs of particles of different size in the chain. Independence of  $\sigma^y$  from the coordinate  $z$  stems from the homogeneity of the strain  $\varepsilon(t)$  and the rheological properties of the system in the saturated zone;  $F_f(\dot{H}, H)$  is the force of viscous resistance caused by liquid flow in a gap;  $F(H)$  is the force of pair interaction of spherical particles determined by the Deryagin equation [10]:

$$F(H) = \pi R G(H), \quad (9)$$

where  $G(H)$  is the specific energy of interaction of flat particles.

To calculate  $F_f$ , we determine the distribution of pressure  $p$  in a gap. Let the  $z$  axis of a cylindrical coordinate system run through the particle centers and the origin of coordinates lie on the surface of one of them. For high-concentration disperse systems the shortest distance between particles is much smaller than their radius ( $H \ll R$ ), and therefore the radial velocity  $V_r$  is much higher than the axial one  $V_z$  ( $V_r \gg V_z$ ),  $\partial p / \partial r \gg \partial p / \partial z$ ,  $\partial V_r / \partial z \gg \partial V_r / \partial r$ . With such simplifications the axisymmetric liquid flow in a gap is described by the equations [11, 12]

$$\mu \frac{\partial^2 V_r}{\partial z^2} = \frac{\partial p}{\partial r}, \quad \frac{\partial p}{\partial z} = 0, \quad \frac{1}{r} \frac{\partial (r V_r)}{\partial r} + \frac{\partial V_z}{\partial r} = 0. \quad (10)$$

The boundary conditions are as follows:

$$z = 0: V_r = 0, \quad V_z = 0, \quad z = H_+: V_r = 0, \quad V_z = -u, \quad r = R: p = p_0, \quad (11)$$

where  $u = -\dot{H}$  is the velocity of approach of particles;  $p_0$  is the outer pressure,  $H_+ = H + r^2/R$  is the distance between spherical surfaces.

Solving problem (10), (11) for the pressure, we find that

$$p(r) - p_0 = \frac{3\mu u R}{2} \left( \frac{1}{H_+^2} - \frac{1}{H_e^2} \right), \quad (12)$$

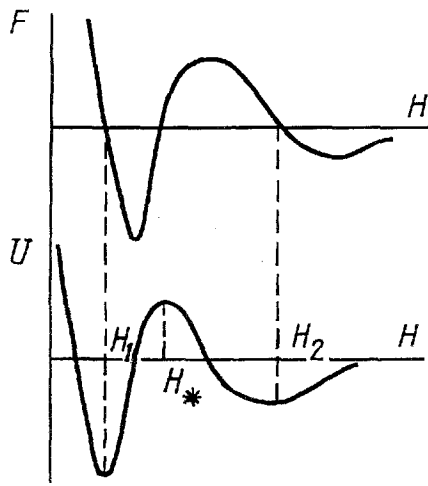


Fig. 1. Isotherms of the interaction forces of spherical particles  $F(H)$  and their interaction energy  $U(H)$ .

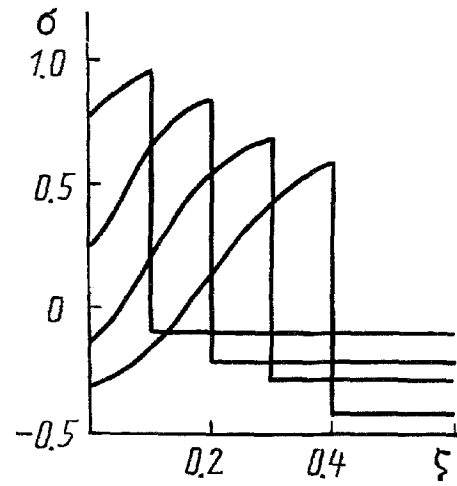


Fig. 2. Plate stresses for different positions of the evaporation front.  $\sigma$ ,  $\text{Pa} \cdot 10^3$ .

where  $H_e = H + R$ . Integrating (12), we obtain the force of viscous resistance

$$F_f(u, H) = \pi \int_R^0 [p(r) - p_0] dr^2 = (3/2) \pi \mu u R^2 \times$$

$$\times \int_H^{H_+} (H_+^{-2} - H_e^{-2}) dH_+ = (3/2) \pi \mu u R^2 (H_e - H)^2 / (H_e^2 H) \approx (3/2) \pi \mu R^2 u / H. \quad (13)$$

The last equality is written with account for the condition  $H \ll H_e$ .

We now consider the function  $G(H)$  in (9) in detail. Its behavior is determined, as a rule, by two components of the surface forces, namely, molecular attraction and electrostatic repulsion. For instance, for quartz particles in a 1-1 aqueous solution of an electrolyte with a concentration  $C = 10^{-1}$  mole/m<sup>3</sup> [13]:

$$G(H) = 64CNkT\gamma\kappa^{-1} \exp(-\kappa H) - B_0/(3H^3),$$

where  $B_0 = 2 \cdot 10^{-29}$  J·m is the constant of dispersion forces,  $\gamma = 0.905$ .

The energy of particle interaction is given by the expression

$$U(H) = \int_H^\infty F(H) dH.$$

The characteristic form of the functions  $U(H)$  and  $F(H)$  is shown in Fig. 1. A number of important features of the rheological behavior of the system in the saturated region are associated with the fact that the interaction energy  $U(H)$  has two potential wells ( $H = H_1, H_2$ ) whose depths differ by several orders of magnitude. The wells are separated by an appreciable barrier [14] whose maximum height is attained at  $H = H_*$ . A pair of particles, with an interlayer thickness between which  $H > H_*$ , approach by overcoming the repulsive force ( $F > 0$ ). When  $H = H_*$  is achieved, the particles penetrate into a very narrow and deep potential well and are fixed there at  $H = H_1 \approx H_*$ . Thus, in relation to the thickness of the liquid interlayer two types of contacts are possible in a linear chain. The interlayers with  $H > H_*$  realize a viscoelastic contact, and the links with  $H = H_*$  a rigid one. Since  $F$  and  $F_f$  depend, according to (9) and (13), on  $R$ , the approach of each particle pair proceeds, by virtue of (8), in a different manner: when  $F$  and  $F_f$  are proportional to  $R$  and  $R^2$ , larger particles approach each other more slowly and attain  $H = H_*$  later. At each moment of time a critical radius  $R_*$  exists such that a pair of particles with

$R < R_*$  resides in the nearest deep potential well and has a rigid contact, while particles with  $R > R_*$  are in a viscoelastic contact. We determine  $R_*$  as

$$R_*(\xi^0) = \max \{ R \mid H(R) = H_* \}. \quad (14)$$

During drying, the number of rigid contacts at a macropoint changes, and their concentration is expressed by the relation

$$\eta(R_*) = 1 - \int_{R_*}^{R_+} \varphi(R) dR. \quad (15)$$

When the evaporation front passes through a macropoint, the structure of the latter is fixed, and rigid contacts turn into condensation contacts of the dry region. Their concentration determines the mechanical properties of the material. The possibility of occurrence of new condensation contacts in the evaporation zone in rapid drying may be neglected because of the small duration of the contracting forces. As the evaporation front penetrates farther,  $\eta$  increases, and therefore the material structure of the dry region is inhomogeneous over to its thickness. The elasticity modulus  $E(\eta)$  increases along with  $\eta$  in the direction from the layer surface to its center.

Thus, based on Eqs. (5), (8), (9), (13), we may represent the rheological relations for different zones:

$$\sigma^e(\xi, t) = \frac{E(\xi)}{1-\nu} [\varepsilon(t) - \varepsilon^0(\xi)], \quad 0 < \xi < \xi^0, \quad (16)$$

$$\sigma^v(t) = \frac{3\pi\mu R^2}{2\langle S \rangle H} \frac{dH(R, t)}{dt} - \frac{\pi R G(H)}{\langle S \rangle}, \quad \xi^0 < \xi < 1. \quad (17)$$

Passing, in (16), (17), from the time variable  $t$  to the coordinate  $\xi^0$  of the evaporation front, we may write in accordance with (1)

$$\xi^0 = \tau^{1/2}, \quad \tau = t/t_1, \quad d\xi^0/d\tau = (2\xi^0)^{-1},$$

whence

$$\sigma^e(\xi, \xi^0) = \frac{E(\xi)}{1-\nu} [\varepsilon(\xi^0) - \varepsilon^0(\xi)], \quad 0 < \xi < \xi^0, \quad (18)$$

$$\sigma^v(\xi^0) = \frac{3\pi\mu R^2 (2\xi^0)^{-1}}{2\langle S \rangle H t_1} \frac{dH(\xi^0)}{d\xi^0} - \frac{\pi R G(H)}{\langle S \rangle}, \quad \xi^0 < \xi < 1. \quad (19)$$

The total tensile forces in the layer in the absence of external forces on its contour are equal to zero:

$$\int_0^{\xi^0} \sigma^e(\xi, \xi^0) d\xi + \int_{\xi^0}^1 \sigma^v(\xi^0) d\xi = 0. \quad (20)$$

Substituting (18), (19) into (20), we arrive at a nonlinear integro-differential equation relative to  $H(\xi^0)$ :

$$-(1 - \xi^0) \left[ \frac{3\pi\mu R^2 (2\xi^0)^{-1}}{2\langle S \rangle H t_1} \frac{dH(\xi^0)}{d\xi^0} - \frac{\pi R G(H)}{\langle S \rangle} \right] =$$

$$= \int_0^{\xi^0} \frac{E(\xi)}{1-\nu} [\varepsilon(\xi^0) - \varepsilon^0(\xi)] d\xi, \quad \xi^0 \in (0, 1), \quad R \in (R_-, R_+). \quad (21)$$

This equation includes both the micro- ( $H, R$ ) and macroparameters ( $\varepsilon, \varepsilon^0$ ). The former characterize the state of an individual link and change within the limits of a chain (macropoint), and the latter pertain to the ensemble as a whole. To establish the relationship between the micro- and macroquantities, we introduce the mean value of the interlayer thickness:

$$\langle H \rangle = \int_{R_-}^{R_+} H(R) \varphi(R) dR, \quad (22)$$

where  $R_-$  is the minimum radius of the particles in a chain. For small strains ( $\langle R \rangle \gg \langle H \rangle$ ) we may write

$$\varepsilon = \frac{\langle H \rangle - \langle H \rangle_0}{\langle R \rangle}, \quad (23)$$

the subscript "0" indicates the onset of drying ( $t = 0$ ).

Prior to solving Eq. (21), we express the shrinkage in terms of  $\langle H \rangle$ . The first term in (7) is given by formula (23). The second summand may be represented as

$$\varepsilon^e(\xi) = \theta f(V),$$

where  $\theta = -\langle H \rangle / \langle R \rangle$ ,  $f(V)$  is a function of the evaporation front velocity.

It is clear that the higher the velocity  $V$ , the shorter the time of residence and the smaller the shrinkage of a macroparticle of the medium in the evaporation zone. At  $V = 0$  the shrinkage is at its maximum and  $\varepsilon^e = \theta$ , and at  $V = \infty$ ,  $\varepsilon^e = 0$ . All these properties are satisfied by the simple relation  $f(V) = V_0 / (V + V_0)$ , which is sufficient to elucidate the qualitative properties of the solution of the problem under consideration. As a result, the shrinkage is expressed as follows

$$\varepsilon^0(\xi) = \frac{\langle H \rangle(\xi) - \langle H \rangle_0}{\langle R \rangle} - \frac{\langle H \rangle(\xi)}{\langle R \rangle} \frac{\xi}{\xi + S}, \quad (24)$$

where  $S = h / (2t_1 V)$  is a dimensionless parameter.

The initial condition for Eq. (21) is obtained from Eq. (8) for the stationary state before the onset of drying when the layer is completely saturated and the capillary forces  $F_k$  act at its surface ( $z = \pm h$ ). Here the viscous friction  $F_f$  is absent and  $F_k$  plays the role of external forces  $F_e$ . Thus, the initial distribution  $H_0(R)$  in the chain is determined by the equality

$$F_k = \pi R G(H_0). \quad (25)$$

**2. Numerical Implementation. Calculation Results.** Equation (21), relations (22)-(24), and initial condition (25) represent a Cauchy problem relative to  $H(\xi^0; R)$ , in which  $R$  is the internal parameter of the chain (ensemble). Solving this problem numerically, we determine the distribution  $H(\xi^0, R)$ , the mean interlayer thickness  $\langle H \rangle$ , and the concentration  $\eta$  of rigid contacts in the ensemble and then evaluate the stresses in the plate using Eqs. (18), (19).

We construct an algorithm for the solution with account for the interrelation of micro- and macroparameters in Eq. (21). In the intervals of variation of the position of the evaporation front  $\xi^0 \in (0, 1)$ , the space coordinate  $\xi \in (0, 1)$ , and the parameter  $R \in (R_+, R_-)$  we introduce the uniform grids  $\{\xi_i^0, i = 1, N; \xi_n, k = 1, N; R_j, j = 1, M\}$  with the steps  $h^0$  and  $r$ , respectively. The grid functions at the nodes are denoted by  $H_i^j = H(\xi_i^0, R_j)$ ,  $\langle H \rangle_i = \langle H \rangle(\xi_i^0)$ ,  $\sigma_{ki}^e = \sigma^e(\xi_k, \xi_i)$ , ... . Then the Euler method for this problem may be written in the form

$$H_{i+1}^j = H_i^j + h^0 \Phi(\zeta_i^0, H_i^j, \langle H \rangle_i, R_j), \quad H^j(0) = H_0^j, \quad (26)$$

$$\Phi = \left\{ \frac{-1}{1 - \zeta_i^0} \int_0^{\zeta_i^0} \frac{E(\eta)}{1 - \nu} \left[ \frac{\langle H \rangle(\zeta_i^0)}{\langle R \rangle} - \frac{\langle H \rangle(\zeta)}{\langle R \rangle} \frac{S}{\zeta + S} \right] d\zeta + \right. \\ \left. + \frac{\pi R_j G(H_i^j)}{\langle S \rangle} \left( \frac{3\pi\mu R_j^2}{4 \langle S \rangle H_i^j \zeta_i^0} \right)^{-1} \right.$$

The initial distribution  $H(R)$  is found from implicit equation (25) by the iterative technique, and the mean  $\langle H \rangle_0$ , the critical radius  $R_{*0}$ , and the concentration  $\eta_0$  are determined by formulas (22), (14), (15). Thus, the right-hand side of Eq. (26) is completely determined and we may evaluate  $H_1^j$ , then  $\langle H \rangle_1$ ,  $R_{*1}$ ,  $\eta_1$ , and so on. At each step in  $\zeta^0$  the algorithm ensures calculation of the microparameters  $\langle H \rangle_i$ ,  $R_{*i}$ , and  $\eta_i$ , in addition to  $H_i^j$ .

In the numerical calculations the following values of the physical constants have been assumed:  $\langle R \rangle = 7 \cdot 10^{-7}$  m,  $\langle \sigma \rangle = 1.67 \cdot 10^{-7}$  m,  $R_- = 2 \cdot 10^{-7}$  m,  $R_+ = 12 \cdot 10^{-7}$  m,  $\mu = 500$  mPa·sec,  $t_1 = 5$  sec,  $S = 0.02$ . Here, a linear dependence of the elasticity modulus on the concentration is assumed

$$E(\eta) = E_0 [1 + \eta(\zeta)],$$

where  $E_0 = 5 \cdot 10^4$  N/m<sup>2</sup> is the elasticity modulus at  $\eta = 0$ .

Calculations are performed up to the evaporation front position  $\zeta_*^0$ , where all particles reside in the nearest deep potential well ( $\eta = 1$ ). In this critical state the viscous properties of the medium in the saturated region disappear, and the rheological behavior of the material at  $\zeta \in (\zeta_*^0, 1)$  is described, in accordance with (16), by the relation

$$\sigma = E(1) (1 - \nu)^{-1} [\varepsilon - \varepsilon^0(\zeta_*^0)]$$

independently of the position of the evaporation front in the indicated interval. Further the stress profile  $\sigma(\zeta)$  is also independent of the position of the front. At a sufficiently high value of  $E(1)$  it coincides with the profile of the critical state.

Figure 2 shows the stresses in the plate as a function of the position of the evaporation front  $\zeta^0$ . In the central region, compression occurs ( $\sigma^y = 0$ ) and the stresses increase as the evaporation front penetrates farther. In the dry region, the dependence of the shrinkage on the drying rate causes an alternating stress and development of extension and contraction zones. The structure of the dried medium is inhomogeneous over its thickness, and the material density and strength increase from the periphery to the center. With increase in the viscosity of the disperse medium, the depth of the evaporation front  $\zeta_*^0$  at the critical position increases.

## NOTATION

$A$ , coefficient in Eq. (1);  $C$ , electrolyte concentration;  $[D]$ , effective diffusion coefficient of the vapor in the porous medium;  $E, E_0$ , Young's moduli;  $F, F_k, F_f, F_e$ , surface and capillary forces, force of viscous resistance, and force acting on the macropoint in the saturated region, respectively;  $H, H_0$ , current and initial shortest distances between spherical particles;  $H_+, H_e$ , distance between the surfaces of spherical particles and its maximum value, respectively;  $H_1, H_2$ , equilibrium distances corresponding to the near- and far-range minima of the interaction energy of particles;  $H_*$ , critical distance corresponding to the energy maximum ( $\partial U/\partial H = 0$  at  $H = H_*$ ,  $H_1 < H_* < H_2$ );  $\langle H \rangle, \langle H \rangle_0$ , mean distance, with respect to the macropoint, between particles and its initial value, respectively;  $h$ , step along the spatial coordinate;  $k$ , Boltzmann constant;  $M$ , molecular mass of the vapor;  $N$ , Avagadro's number;  $p, p_0$ , liquid pressure in an interparticle gap and external pressure, respectively;  $P_s$ , pressure

of the saturated vapor;  $R$ , universal gas constant, particle radius;  $R_+$ ,  $R_-$ ,  $R_*$ ,  $\langle R \rangle$ , maximum, minimum, critical, and mean particle radius, respectively;  $r$ , axis in the cylindrical coordinate system;  $S$ , dimensionless parameter;  $\langle S \rangle$ , averaged area of the cell;  $T$ , temperature;  $t$ ,  $t_1$ , current time and time of complete layer drying, respectively;  $U$ , free energy of interaction of particles;  $V$ , velocity of the evaporation front;  $V_r$ ,  $V_z$ , radial and axial velocity, respectively;  $x$ ,  $y$ ,  $z$ , Cartesian coordinate system;  $j$ , constant;  $\delta_{ij}$ , Kronecker symbol;  $\varepsilon_{ij}$ ,  $\varepsilon_{ij}^0$ , components of the strain and shrinkage tensor;  $\varepsilon$ ,  $\varepsilon^0$ , strain and shrinkage along the  $x$  (or  $y$ ) axis;  $\varepsilon^e$ , shrinkage in the evaporation zone;  $\xi$ ,  $\xi^0$ ,  $\xi_*^0$ , dimensionless spatial coordinate, coordinate of the position of the evaporation front, and its critical value;  $\eta$ , concentration of rigid contacts;  $\theta$ , maximum shrinkage in the evaporation zone;  $\kappa$ , inverse Debye radius;  $m$ , liquid viscosity;  $\nu$ , Poisson coefficient;  $\rho$ , liquid density;  $\sigma_{ij}$ , components of the stress tensor;  $\sigma^y$ ,  $\sigma^e$ , normal component of the stress tensor along the  $x$  (or  $y$ ) axis in the viscoelastic and elastic regions;  $\langle \sigma \rangle^2$ , dispersion;  $\tau$ , dimensionless time;  $\varphi$ , density of the size distribution of particles.

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